Ch. 2 Elasticity

Class: B. Sc. I Semester Physics-Paper I

Deformation: The change in shape or size or both in shape and size of the body by application of force is called as deformation.

Deforming Force: The force which is responsible to produce deformation is called as deforming force.

Elasticity: The property possessed by a material body by virtue of which, it opposes a change in its shape and size and regains its original dimensions as soon as the deforming forces are removed is called elasticity.

- Bodies which can recover completely their original dimensions, on the removal of the deforming forces are said to be perfectly elastic.
- On the other hand, bodies which do not show any tendency to recover their original condition, are said to plastic body.
- There are no perfectly elastic or plastic bodies. The nearest approach to a perfectly elastic body is **quartz fiber** and perfectly plastic body is **putty.**
- We consider the bodies which are Homogeneous and isotropic i.e. which have the same properties at all points and in all directions. For, these have similar elastic properties in every direction.
- Fluids (i.e. liquids and gases), some of which may exhibit different properties at different points and in different directions i.e. may be heterogeneous and anisotropic.

Stress and its types:

When a body is deformed, internal forces are set up within the body, trying to restore it to its original dimensions.

- These forces are directly proportional to the deformation. These internal forces restore the body to its original dimensions, when deforming forces are removed.
- The internal restoring force per unit area is called the stress.

$$Stress = \frac{Internal\ Restoring\ Force}{Area}$$

- The magnitude of internal restoring force must be equal to that of applied force but opposite in direction.
- $Stress = \frac{Applied\ Force}{Area}$

The dimensions of stress are those of pressure i.e. $[ML^{-1}T^{-2}]$ and SI unit is N/m^2 , CGS unit is $dyne/cm^2$.

There are three types of stress:

i) Tensile or longitudinal stress

ii) Volume stress and

iii) Shearing stress

i) **Tensile or longitudinal stress:** When the deformation consists of a change in length, the stress is called tensile or longitudinal stress.

If a wire is suspended from a fixed support and mass M is attached to its free end, a force **F = Mg** acts on wire.

Let r is radius of the wire, its area of cross-section is $\mathbf{A = \pi r^2}$.

$$Longitudinal\ Stress = \frac{applied\ force}{area} = \frac{F}{A} = \frac{Mg}{\pi r^2}$$

ii. Volume stress: When the deformation consists of a change in volume, the stress is called volume stress.

- The change in volume is produced by increasing or decreasing the pressure on the body.
- If dP is the change in the pressure,
- $Volume\ Stress = \frac{applied\ force}{area} = Change\ in\ Pressure = dP$

iii. Shearing stress: When the deformation consists of a change in shape, the stress is called the shearing stress.

- To produce a change in shape, a force is applied tangentially to some area of the body.
- If F is the force applied tangentially to an area A,
- $Shearing\ Stress = \frac{applied\ force}{area} = \frac{F}{A}$

Strain and its types

Strain: When a body is acted upon by the forces in equilibrium, the body gets deformed. The deformation consists of a change in the dimensions of the body.

- The change in dimensions per unit original dimensions is called the strain.
- Strain is a ratio of two similar quantities, it has no units.

There are three types of strain:

i) Tensile or longitudinal strain

ii) Volume strain

iii) Shearing strain

i. Tensile or longitudinal strain:

When the deformation consists of change in length of the body, the strain is called longitudinal or tensile strain.

Let a cylindrical wire or rod of original L undergo a change in length l *as shown in fig. a*, then

$$Longitudinal\ Strain = \frac{Change\ in\ length}{original\ length} = \frac{l}{L}$$

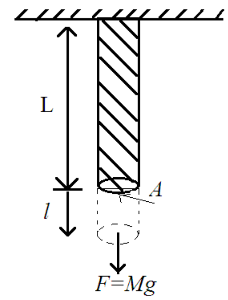


Fig. (a) Deformation in length

ii. Volume strain:

If the deforming forces produce a change in volume of the body, the strain is called volume strain.

- If the original volume V changes by an amount dV *as shown in fig. b*

$$Volume\ Strain = \frac{Change\ in\ volume}{original\ volume} = \frac{dV}{V}$$

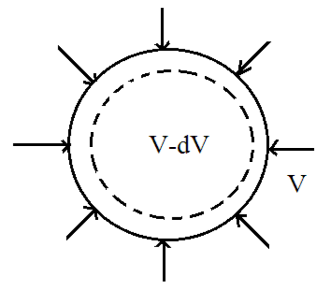


Fig. (b) Deformation in volume

iii. Shearing strain:

If the applied force produces a change in the shape of the body, the strain is called shearing strain.

- Consider a rectangular surface ABCD whose lower side CD is kept fixed and tangential force F is applied to upper side AB.
- Due to the force, the shape of rectangular changes to parallelogram A′B′CD.

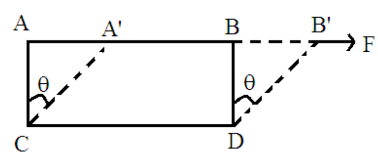


Fig. (c) Deformation in Shape

- The deformation is due to lateral displacement of the layers AB in the direction of force.

Let AA′ is lateral displacement of layer AB then

$$Shearing\ Strain = \frac{Lateral\ displacement\ of\ any\ layer}{distance\ of\ it\ from\ fixed\ layer}$$

$$Shearing\ strain = \frac{AA'}{AC} = tan\theta$$

Since θ is usually very small then $tan\theta \cong \theta$

$$Shearing\ strain = \theta$$

Elastic limit: The maximum stress to which a body can be subjected without permanent deformation is called its elastic limit.

- Within the elastic limit, the body is perfectly elastic and beyond the elastic limit body is permanently deformed.

Modulus of elasticity or elastic constant:

- The modulus of elasticity is defined as the ratio of stress to strain.

$$Modulus\ of\ elasticity = \frac{stress}{strain}$$

- SI unit of modulus of elasticity is N/m^2 and its CGS unit is $dyne/cm^2$. its dimensions are $[M^1L^{-1}T^{-2}]$

Types of modulus of elasticity:

There are three types of modulus of elasticity;

i) Young's modulus (Y)

ii) Bulk modulus (K)

iii) Modulus of rigidity (η)

i). Young's modulus (Y):

The ratio of longitudinal stress to longitudinal strain, within the elastic limit is called as Young's modulus. It is denoted by Y.

Let *F* be the force applied normally on a wire of cross-sectional area *A*, then the stress is *F/A*.

If there be change in length *l* produced in the original length L, the strain is *l/L*.

$$Young'smodulus = \frac{Longitudinal\ stress}{Longitudinal\ strain}$$

$$Y = \frac{F/A}{l/L} = \frac{FL}{Al}$$

If F= mg be load applied on wire of cross-sectional area πr^2 then,

$$Y = \frac{mgL}{\pi r^2 l}$$

If *L*=1, *A*=1 and *l*=1, we have Y = F.

If a material of unit length and unit area of cross-section could be pulled so as to increase in length by unity, i.e. to double its length, the force applied measure would measure the value of Young's modulus.

Young's modulus is the property of solids only.

ii). Bulk modulus (K)

The ratio of volume stress to volume strain, within the elastic limit is called as Bulk modulus. It is denoted by K.

Suppose that a body of volume V is subjected to an additional pressure dP. The increase in pressure will produce a decrease in its volume. If the volume decreases by an amount dV.

Therefore, Volume stress = dP, *Volume strain* = dV/V

$$Bulk\ Modulus = \frac{Volume\ stress}{Volume\ strain}$$

$$K = \frac{dP}{dV/_{V}} = V\frac{dP}{dV}$$

Bulk modulus is referred to as incompressibility and hence its reciprocal is compressibility; so, compressibility of a body is equal to 1/K, where K is its Bulk modulus.

Bulk modulus is possessed by solids, liquids and gases.

iii). Modulus of rigidity (η)

The ratio of shearing stress to shearing strain, within the elastic limit is called as modulus of rigidity. It is denoted by η.

Suppose that lower surface of rectangle is kept fixed while tangential force F is applied to the upper surface. If A is the area of rectangle.

Shearing stress = F/A, Shearing strain = tanθ =θ

$$Modulus\ of\ rigidity = \frac{Shearing\ stress}{Shearing\ Strain}$$

$$Modulus\ of\ rigidity = \frac{F/_{A}}{\theta} = \frac{F}{A\theta}$$

Modulus of rigidity is the property of solids only.

The relation between the three elastic constants is,

$$\frac{3}{\eta} + \frac{1}{K} = \frac{9}{Y}$$

Twisting Couple on a cylindrical rod or wire:

Consider a cylindrical rod of length l and radius r, of a material of coefficient of rigidity η.

Let its upper end be fixed and a couple be applied to the lower end in a plane perpendicular to its length twisting it through an angle θ.

In the equilibrium position, the twisting couple is equal and opposite to the restoring couple. Let us calculate the value of this couple.

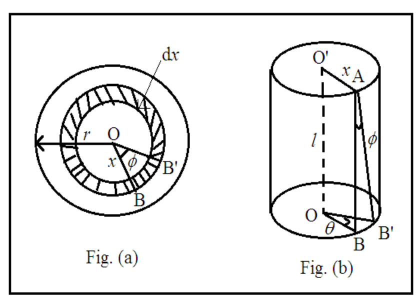


Fig. (a) Fig. (b)

Imagine the cylinder consists of a large number of co-axial, hollow cylinders and consider one such hollow cylinder of radius x and thickness dx.

Let AB be a line parallel to the axis as shown in fig (b), before the cylinder is twisted. On twisting, since points B shifts to B′, the line takes the position AB′

The angle through this hollow cylinder is sheared is $\angle BAB' = \phi$.

$$\text{Then } BB' = l.\phi. \qquad \text{Also, } BB' = x.\theta$$

$$\therefore \quad l.\phi = x.\theta$$

$$\therefore \quad \phi = x\theta / l$$

Obviously, ϕ will have the maximum value where x is the greatest i.e. the maximum strain is on the outermost part of the cylinder and least on the innermost. The shearing stress is not uniform all through. Thus, although the angle of shear is the same for any one hollow cylinder, it is different for different cylinders.

We know that, coefficient of rigidity is,

$$\eta = \frac{Shearing\ stress}{strain\ or\ angle\ of\ shear} = \frac{F}{\phi}$$

$$F = \eta.\phi = \frac{\eta x\theta}{l}$$

Face area of this hollow cylinder= $2\pi x.dx$

Total shearing force on this area is,

$$= 2\pi x dx \times \frac{\eta x\theta}{l} = 2\pi\eta\frac{\theta}{l}.x^2 dx$$

Moment of this force about the axis OO′ of the cylinder is

$$= 2\pi\eta\frac{\theta}{l}.x^2 dx.x = \frac{2\pi\eta\theta x^3 dx}{l}$$

Integrating this expression between the limits $x=0$ and $x=r$

We have,

Total twisting couple on the cylinder,

$$= \int_0^r \frac{2\pi\eta\theta x^3 dx}{l} = \frac{2\pi\eta\theta}{l}\int_0^r x^3 dx$$

$$= \frac{2\pi\eta\theta}{l}\left[\frac{x^4}{4}\right]_0^r = \frac{2\pi\eta\theta}{l}\left[\frac{r^4}{4}\right]$$

$$= \frac{\pi\eta\theta r^4}{2l}$$

If $\theta = 1$ radian, we have, twisting couple per unit twist of the cylinder is

$$C = \frac{\pi\eta\theta r^4}{2l}$$

This twisting couple per unit twist of the wire is also called the torsional rigidity of the cylinder or wire.

Bending of a beam

A beam is a rod or a bar of uniform cross-section (circular or rectangular) of homogeneous, isotropic elastic material, whose length is very compared with its thickness.

When such a beam is fixed at one end and loaded at the other within the limit of perfect elasticity, the loaded end sinks a little. As a result, the beam gets bent under the action of couple due to the load applied as shown in fig. (a).

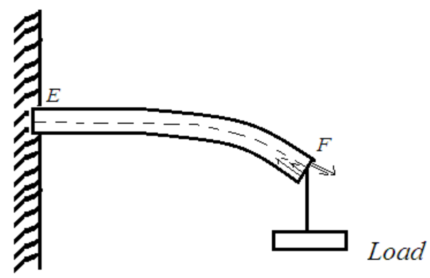


Fig (a)

The upper surface of the beam gets stretched and assumes a convex form and its lower surface gets compressed and assumes a concave form.

A beam can be supposed to be made of very thin rods completely packed on each other. Such rods are called as filaments Hence in the diagram filaments are shown parallel to each other. Direction of central filament AB indicates neutral axis of beam. If we consider surface of beam drawn perpendicular to diagram along neutral axis of the rod that surface is called neutral surface. The beam bent to form an arc of a circle as shown in fig. (b).

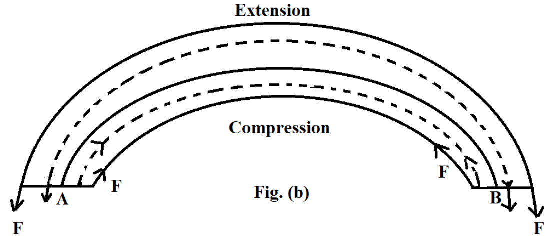
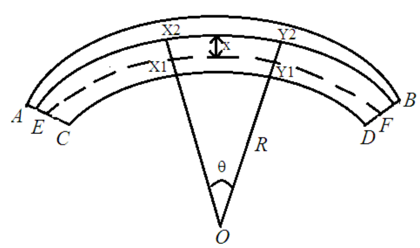


Fig. (b)

In this process all filaments above neutral axis are stretched or extended but the filaments below neutral axis is compressed. Thus, tensile or longitudinal strains are produced in filament. The strain in uppermost filament as well as lowermost filament is maximum. The filament along neutral axis AB is neither extended or contracted in the process of bending. Thus, it remains neutral and it is termed as neutral axis.

Expression of Bending moment of a beam:



Consider a small part of the beam be bent in the form of a circular arc, subtending an angle θ at the centre of curvature O as shown in figure.

Let R be the radius of curvature.

Let EF be neutral axis of the beam.

Let X_1Y_1 be a point on the neutral axis.

Let X_2Y_2 be points on the filament at a distance x from the neutral axis.

$\angle X_1OY_1 = \theta$ From Figure, $X_1Y_1 = R\theta$ and $X_2Y_2 = (R+x)\theta$

Since, original length is X_1Y_1

$\therefore$Extension on $X_1Y_1 = X_2Y_2 - X_1Y_1 = (R+x)\theta - R\theta = x\theta$

$$Tensile\ or\ linear\ strain = \frac{extension}{initial\ length} = \frac{x\theta}{R\theta} = \frac{x}{R}$$

i.e. the strain is proportional to the distance from the neutral axis. This strain is produced due to stress applied on it.

Let δa be area of cross section of filament

$$Tensile\ stress = \frac{F}{\delta a}$$

We know that, Young's modulus of material of beam is,

$$Y = \frac{tensile\ stress}{tensile\ strain} = \frac{F/\delta a}{x/R} = \frac{FR}{x\delta a}$$

$$F = \frac{Yx\delta a}{R}$$

This is restoring force acting on area δa.

The moment of force about neutral axis = $F.x$

$$= \frac{Yx\delta a}{R}.x = \frac{Yx^2\delta a}{R}$$

The moments of the forces acting on both the upper and the lower halves of the section are in the same direction. The total moment of the forces acting on the filaments is given by

$$M = \sum \frac{Yx^2\delta a}{R} = \frac{Y}{R}\sum \delta a x^2$$

Where $\Sigma\delta a.x^2 = I_g$ is the geometrical moment of inertia of beam.

$$Bending\ moment\ of\ beam\ M = \frac{YI_g}{R}$$

This is the expression of restoring couple or bending moment of the beam.

<u>Cantilever:</u>

A cantilever is a beam fixed horizontally at one end and loaded at the other.

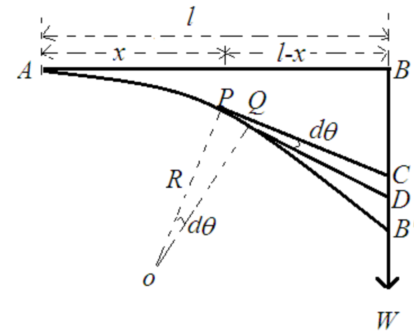
Here, two cases arise

i) When the weight of the is ineffective,

ii) When weight of the beam is effective.

i) When weight of the beam is ineffective:

Let AB represent the neutral axis of a cantilever of length *l* fixed at the end *A* and loaded at *B* with a weight *W*, such that the end B is deflected or depressed into the position B′ and neutral axis takes up the position AB′.



It being assumed that the weight of the beam itself produces no bending.

Consider a section P of the beam at a distance x from the fixed end A.

The moment of the external couple at this section, due to load W or the bending moment acting on it = $W \times PB' = W(l-x)$

We know that, bending moment of the beam = $Y.\ Ig/R$

Since, beam is in equilibrium then

$$W(l-x) = Y.\ Ig/R \text{ ----------------- } (1)$$

For a point Q, at a small distance *dx* from P, So that

$$PQ = R.\ d\theta$$

$$dx = R.\ d\theta \ \ or\ R = dx/d\theta$$

Substituting value of R in equation (1), we have

$$W(l-x) = \frac{YI_g d\theta}{dx}$$

$$d\theta = \frac{W(l-x)dx}{YI_g} ------(2)$$

Draw tangents to the neutral axis at P and Q meeting the vertical line through BB’ in C and D respectively. Then the angle subtended by them is also equal to $d\theta$.

Now, the depression of Q below P is equal to CD, equal (say *dy*)

Then,

$$dy = (l-x)d\theta$$

By substituting value of dθ from eq^n (2),

$$dy = (l-x)\frac{W(l-x)dx}{YI_g}$$

$$= \frac{W(l-x)^2 dx}{YI_g} ----(3)$$

∴ The depressions y=BB′ of the loaded end B below the fixed end A, is obtained by integrating the expression for *dy* between the limits x=0 and x= *l*.

$$y = \int_0^l \frac{W(l-x)^2 dx}{YI_g} = \frac{W}{YI_g}\int_0^l (l-x)^2 dx$$

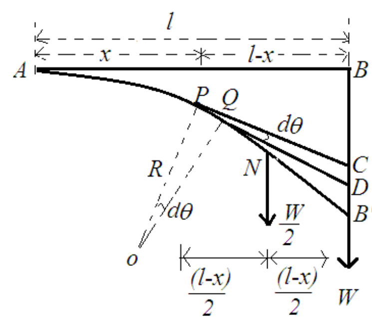
$$y = \frac{W}{YI_g}\left[\frac{(l-x)^3}{3}\,(-1)\right]_0^l = -\frac{W}{3YI_g}[0-l^3]$$

$$y = \frac{Wl^3}{3YI_g} ------ (4)$$

Eq^n (4) represent the expression of depression of beam of cantilever, when weight of beam is ineffective.

ii) <u>When weight of the beam is effective:</u>

Let AB represent the neutral axis of a cantilever of length *l* fixed at the end *A* and loaded at *B* with a weight *W*, such that the end B is deflected or depressed into the position B′ and neutral axis takes up the position AB′.



It being assumed that the weight of the beam itself produces bending.

Let W_1 be weight per unit length of cantilever i.e. weight of cantilever of lentgh *l* is $\frac{W_1}{l}$

Suppose this weight acting at midpoint N of the portion *(l-x)* of beam.

The weight $\frac{W_1(l-x)}{l}$ is acting at a distance $\frac{(l-x)}{2}$ from the section PQ.

The bending moment of length of beam $\frac{(l-x)}{2}$ *is given by*

$$= \frac{W_1(l-x)}{l}.\frac{(l-x)}{2} = \frac{W_1(l-x)^2}{2l}$$

The total bending moment of the beam = $W(l-x) + \frac{W_1(l-x)^2}{2l}$

Since, the beam is in equilibrium, this must be equal to $\frac{YI_g}{R}$

$$\therefore \quad \frac{YI_g}{R} = W(l-x) + \frac{W_1(l-x)^2}{2l} ------ (1)$$

For a point Q, at a small distance *dx* from P, So that

$$PQ = R.\, d\theta$$

$$dx = R.\, d\theta \ \ or \ R = dx/d\theta$$

Substituting value of R in equation (1), we have

$$\therefore\ W(l-x)+\frac{W_1(l-x)^2}{2l}=\frac{YI_g d\theta}{dx}$$

$$\therefore\ d\theta=\frac{dx}{YI_g}\left[W(l-x)+\frac{W_1(l-x)^2}{2l}\right]----(2)$$

Draw tangents to the neutral axis at P and Q meeting the vertical line through BB' in C and D respectively. Then the angle subtended by them is also equal to $d\theta$.

Now, the depression of Q below P is equal to CD, equal (say dy)

Then $\quad dy = (l-x).d\theta$

By substituting value of dθ from eq^n (2),

$$dy=(l-x)\frac{dx}{YI_g}\left[W(l-x)+\frac{W_1(l-x)^2}{2l}\right]$$

$$\therefore dy=\left[W(l-x)^2+\frac{W_1(l-x)^3}{2l}\right]\frac{dx}{YI_g}------(3)$$

The depressions $y=BB'$ of the loaded end B below the fixed end A, is obtained by integrating the expression for dy between the limits $x=0$ and $x=l$.

$$y=\int_0^l\left[W(l-x)^2+\frac{W_1(l-x)^3}{2l}\right]\frac{dx}{YI_g}$$

$$\therefore y=\frac{W}{YI_g}\int_0^l(l-x)^2dx+\frac{W_1}{2lYI_g}\int_0^l(l-x)^3dx$$

$$\therefore y=\frac{W}{YI_g}\left[\frac{(l-x)^3}{3}(-1)\right]_0^l+\frac{W_1}{2lYI_g}\left[\frac{(l-x)^4}{4}(-1)\right]_0^l$$

$$\therefore y=\frac{-W}{3YI_g}[0-l^3]+\frac{-W_1}{8lYI_g}[0-l^4]$$

$$\therefore\quad y=\frac{Wl^3}{3YI_g}+\frac{W_1l^3}{8YI_g}$$

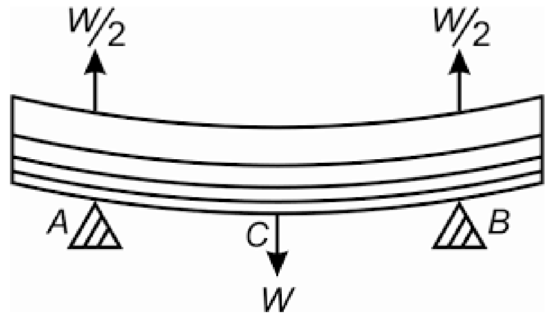
$$\therefore y=\frac{l^3}{YI_g}\left[\frac{W}{3}+\frac{W_1}{8}\right]$$

$$\therefore y=\frac{l^3}{3YI_g}\left[W+\frac{3W_1}{8}\right]------(4)$$

Eq^n (4) gives the depression of beam of cantilever when weight of beam is effective.

This represent that the beam behaves a though it is loaded at the end B with a weight W plus 3/8 th of the weight of the beam.

Depression of a beam loaded at the center:



When a beam is supported near its ends and loaded at the centre, it shows maximum depression at the loaded point. Figure shows a beam supported on two knife-edges indicated by A and B. Suppose that it is loaded in the middle at C with a weight W.

The reaction at each knife edge can be taken to be (W/2) in the upward direction. In this position, the beam may be considered as equivalent to two inverted cantilevers, fixed at C.

The bending in these two cantilevers will be produced by the reaction load – acting upwards at A and B.

Let L be the length of the beam, the length of each cantilever (Ac and BC) is L/2.

The depression of C below A and B is given by relation,

$$y = \frac{(weight) \times (length)^3}{3YI_g} = \frac{{}^{W}\!/_{2} \cdot ({}^{L}\!/_{2})^3}{3YI_g}$$

$$y = \frac{{}^{WL^3}\!/_{16}}{3YI_g} = \frac{WL^3}{48YI_g} ------(1)$$

If the beam be circular cross-section, we have geometrical moment of inertia is

$$I_g = ak^2 = \frac{\pi r^4}{4}$$

Where r is radius of its cross section.

so that, depression of beam is,

$$y = \frac{WL^3}{48Y.\frac{\pi r^4}{4}} = \frac{WL^3}{12\pi Y r^4} --- (2)$$

If beam of a rectangular cross-section of breadth *b* and depth *d*, we have,

$$I_g = ak^2 = \frac{bd^3}{12}$$

so that, depression of beam is,

$$y = \frac{WL^3}{48Y.\frac{bd^3}{12}} = \frac{WL^3}{4Ybd^3} ----- (3)$$

Numericals:

1. A wire of length 3m and diameter 0.6mm is stretched by load of 10kg.wt. If Young's modulus of wire is 2×10^{11} N/m^2. Find i) longitudinal stress, ii) longitudinal strain, iii) increase in length.

   Solution: Given

   L = 3m,

   $2r = 0.6\text{mm} = 0.6\times10^{-3}$ m$=6\times10^{-4}$m

   $r = 3\times10^{-4}$m

   $M = 10kg$

   $Y=2\times10^{11}$ N/m^2

i) Longitudinal Stress = F/A = $\frac{Mg}{\pi r^2}$

$$= \frac{10\times9.8}{3.14\times(3\times10^{-4})^2} = \frac{98}{3.14\times9}\times10^{8} = 3.4678\times10^{8}\ \text{N/m}^2$$

ii) Longitudinal strain=longitudinal stress/Y

$$= \frac{3.4678\times10^{8}}{2\times10^{11}} = 1.7339\times10^{-3}$$

iii) *Strain = l/L*

Increase in length l=Strain ×L$=1.7339\times10^{-3}\times3=5.2017\times10^{-3}$ m

=5.2017mm

2. What pressure should be applied to a lead block to reduce its volume by 10%. (Bulk modulus = 6×10^{9}N/m$^{2.}$

   Solution: Given

   dV=10%V = 10/100 V = 0.1V

   K = 6×10^{9} N/m^2

   dP = ?

   Bulk modulus,

$$K = \frac{VdP}{dV}$$

   Change in pressure is,

$$dP = \frac{K.dV}{V} = \frac{6\times10^{9}\times0.1V}{V} = 6\times10^{8}N/m^2$$

3. What mass must be suspended from free end of steel wires of length 2m and diameter 1mm to stretch it by 1mm? (Young's modulus of steel = 2×10^{11} N/m^2).

   Solution: Given

   $L = 2m,\ l = 1\ mm = 10^{-3}m$

   $2r = 1\ mm$

$r = 0.5\times10^{-3}m = 5\times10^{-4}m$, $Y = 2\times10^{11}$ N/m^2

$m = ?$

Young's Modulus of wire is,

$$Y = \frac{mgL}{\pi r^2 l}$$

$$m = \frac{Y\pi r^2 l}{gL}$$

$$= \frac{2\times10^{11}\times3.14\times25\times10^{-8}\times10^{-3}}{9.8\times2}$$

$$= \frac{78.5}{9.8} = 8.01Kg$$

4. A cube each side of 6cm whose upper surface is applied by a force of 0.36N. As a effect its upper face is displaced laterally by 0.75cm. Calculate shearing strain, stress, and modulus of rigidity.

   Solution: Given

   L = 6cm = 6×10^{-2}m

   F = 0.36 N

   l = 0.75 cm

$$Shearing\ strain = \frac{l}{L} = \frac{0.75}{6} = 0.125$$

$$Shearing\ stree = \frac{F}{A} = \frac{F}{L^2} = \frac{0.36}{36\times10^{-4}} = 0.01\times10^4 = 100N/m^2$$

$$Modulus\ of\ rigidity = \frac{stress}{strain} = \frac{100}{0.125} = 800\ N/m^2$$

5. What couple must be applied to a wire on a meter long, 1mm in diameter in order to twist one end of it through π/2 radians, the other end remaining fixed? The rigidity of material of wire is 2.8×10^{11} dynes/cm^2.

Solution: Given

l = 1m = 100 cm

2r = 1mm

r= 0.5mm = 0.5×10^{-1} cm = 5×10^{-2} cm

θ= π/2 radian, η = 2.8×10^{11} dynes/cm^2

Twisting couple applied on a wire is,

$$C = \frac{\pi\eta\theta r^4}{2l}$$

$$C = \frac{3.14\times2.8\times10^{11}\times\frac{\pi}{2}\times(5\times10^{-2})^4}{2\times100}$$

$$C = \frac{3.14\times1.4\times3.14\times625\times10^3}{200}$$

$$C = \frac{8627.15}{2} \times 10 = 4313.6 \times 10 = 4.313 \times 10^4 dyne - cm$$

6. A brass bar 1cm.square in cross section is supported on two knife edges 100cm apart. A load of 1kg.wt at the centre of the bar depresses that point by 2.51mm. What is the Young's modulus of brass?

Solution: Given

b=d=1 cm, l=100 cm

W= 1kg.wt = 1000×980 dyne = 98×10^4 *dyne*

depression y = 2.51mm = 0.251cm.

We know that, depression of bar at the centre is

$$y = \frac{WL^3}{48YI_g}$$

Where

$$I_g = \frac{bd^3}{12} = \frac{1}{12}$$

Young's modulus of bar is,

$$Y = \frac{WL^3}{48yI_g} = \frac{98 \times 10^4 \times (100)^3}{48 \times 0.251 \times {}^1/_{12}}$$

$$Y = \frac{98 \times 10^{10}}{4 \times 0.251} = 97.6 \times 10^{10} = 9.76 \times 10^{11}\ dynes/cm^2$$

7. A steel rod of 1 meter of length, 2cm in breadth and thickness 5mm is supported horizontally on two parallel knife edges at its ends. If a load of 1 kg at the centre of the rod depresses this point by 4.9mm. Calculate Young's modulus of steel.

Solution: Given

L =1m, b=2cm = 2×10^{-2}m, d=5mm = 5×10^{-3}m

M= 1 kg

y= 4.9mm = 4.9×10^{-3}m

Y = ?

Depression produced in a bar

$$y = \frac{WL^3}{48YI_g}$$

But $I_g = \frac{bd^3}{12} = \frac{2\times10^{-2}\,(5\times10^{-3})^3}{12} = \frac{2\times125\times10^{-11}}{12} = 20.83 \times 10^{-11}$

Young's modulus of steel bar is

$$Y = \frac{MgL^3}{48yI_g} = \frac{1 \times 9.8 \times 1}{48 \times 4.9 \times 10^{-3} \times 20.83 \times 10^{-11}}$$

$$= \frac{9.8}{4899.22} \times 10^{14} = 0.002 \times 10^{14} = 2 \times 10^{11} N/m^2$$

8. I an experiment the diameter of rod was 2cm and the distance between the knife edges 100cms. On putting a load of 900gms. At the middle point depression was 0.025cm. Calculate Young's modulus of the substance.

Solution: Given

2r=2cm, $\therefore$ r = 1cm

L =100cm, m= 900 gm, y = 0.025 cm then Y=?

Depression produced in a bar

$$y = \frac{WL^3}{48YI_g}$$

But, Bar is circular, $I_g = \frac{\pi r^4}{4} = \frac{3.14 \times 1}{4} = 0.785$

Young's modulus of the substance is

$$Y = \frac{MgL^3}{48yI_g} = \frac{900 \times 980 \times (100)^3}{48 \times 0.025 \times 0.785} = \frac{882 \times 10^9}{0.942}$$

$$Y = 936.3 \times 10^9 = 9.363 \times 10^{11} dyne/cm^2$$

Multiple Choice questions:

1. Which of the following body is perfectly elastic?
   a. Rubber **b. Quartz** c. Putty d. Glass
2. The modulus of elasticity is dimensionally equivalent to -----
   a. Stress b. Pressure c. Surface tension **d. Both a and b.**
3. A wire is stretched double its length. The strain is-----
   a. Infinity **b. 1** c. Zero d. 0.5
4. Which of the following quantity is unitless?
   a. Stress b. Young's modulus **c. Strain** d. Pressure
5. The pressure required to reduce the given volume of water by 1% is----

(Bulk modulus of water = $2\times10^9 N/m^2$

a. **$2\times10^7 N/m^2$** b. $2\times10^9 N/m^2$

c. $2\times10^{10} N/m^2$ d. $2\times10^8 N/m^2$

6. When wire is twisted, the strain produced in it is------

   a. Shear b. Longitudinal c. Volumetric d. Tensile

7. Bulk modulus is the property of the-----

   a. Solid b. Liquid c. Gases **d. All of these**

8. Relation between Young's modulus (Y), Bulk modulus (K) and modulus of rigidity (η) is

   a. $\boldsymbol{\frac{9}{Y} = \frac{3}{\eta} + \frac{1}{K}}$ b. $\frac{9}{Y} = \frac{1}{\eta} + \frac{1}{K}$

   c. $\frac{1}{Y} = \frac{3}{\eta} + \frac{1}{K}$ d. $\frac{1}{3Y} = \frac{3}{\eta} + \frac{1}{K}$

9. Twisting couple acting on cylindrical rod or wire is ------

   a. $\frac{\pi\eta\theta r^2}{2l}$ **b.** $\boldsymbol{\frac{\pi\eta\theta r^4}{2l}}$

   c. $\frac{\pi\eta\theta r}{l}$ d. $\frac{\pi\eta\theta r^3}{2l}$

10. In a bending of beam, the upper surface of beam is stretched and assumes ------- form.

    a. concave b. plane **c. convex** d. neutral

11. In bending of beam, neutral axis is the axis at which the beam ---

    a. elongated b . Contracted

    c. neither elongated nor contracted

    d. both elongated and contracted

12. The bending moment of beam is expressed as,----

a. $\frac{YI_g}{2R}$ b. $\frac{RI_g}{Y}$ c. $\frac{Y}{I_g R}$ **d.** $\boldsymbol{\frac{YI_g}{R}}$

13. Depression produced in a beam of cantilever, when weight of beam is ineffective -----

a. $\boldsymbol{\frac{WL^3}{3YI_g}}$ b. $\left[W + \frac{3W_1}{8}\right]\frac{L^3}{3YI_g}$ c. $\frac{2WL^3}{3YI_g}$ d. $\frac{WL^3}{YI_g}$

14. Depression produced in a beam of cantilever, when weight of beam is effective -----

a. $\frac{WL^3}{3YI_g}$ b. $\boldsymbol{\left[W + \frac{3W_1}{8}\right]\frac{L^3}{3YI_g}}$ c. $\frac{2WL^3}{3YI_g}$ d. $\frac{WL^3}{YI_g}$

15. Depression produced in a beam, when beam is loaded at centre is---

a. $\frac{WL^3}{3YI_g}$ b. $\frac{WL^3}{12YI_g}$ c. $\frac{WL^3}{36YI_g}$ **d.** $\boldsymbol{\frac{WL^3}{48YI_g}}$

16. If the beam be of a circular cross-section, geometrical moment of inertia of beam is -----

a. $\pi r^4/4$ b. $\pi r^3/3$ c. $\pi r^2/4$ d. $\pi r/4$

17. If the beam be of a rectangular cross-section of breadth *b* and depth *d* loaded at the centre, the depression produced in a beam is-----

a. $\frac{WL^3}{3Ybd^3}$ b. $\frac{WL^3}{12YI_g}$ **c.** $\boldsymbol{\frac{WL^3}{4Ybd^3}}$ d. $\frac{WL^3}{12Ybd^3}$

18. A wire of length 1m and diameter 1mm is clamped at one of its ends. The couple required to twist the other end by 90^0 is – (modulus of rigidity = 2.8×10^{10} N/m^2)

a. 1.23×10^{-3}N-m b. 2.32×10^{-3}N-m

c. 6.23×10^{-3}N-m **d. 4.32×10^{-3}N-m**

19. A brass bar of rectangular cross-section having breadth 2 cm and depth 6cm then geometrical moment of inertia of bar is,

    a. 216 **b. 36** c. 24 d. 45

20. The modulus of elasticity is the ratio of --------

    a. stress to strain b. Strain to stress

    c. stress to young's modulus d. strain to bulk modulus